A STOCHASTIC PROCESS MODEL OF WORK FORCE HISTORY

P. Krishnan
University of Alberta

1 INTRODUCTION

Working life table is an accepted tool in demography. But this conventional technique has been found to be inappropriate for use with female populations of the developed societies in view of the bimodality of their labor force participation. Garfinkle (1968) has developed a procedure to take care of this feature of the modern female's work force participation. Since this requires refined data, the Garfinkle methodology cannot be put to use in most instances. Terry and Sly (1972) have adapted the working life table technique to get around the problem of bimodality by dividing the stationary work population into three components of those who (a) work continuously (b) work temporarily and (c) are temporarily out of work, and doing separate analysis of each of the components. These may be considered as ad hoc solutions to the bimodality problem. There are other problems as well which beset the labor force analysis. Some of these are pointed out elsewhere in the paper. These problems also require solutions for a better and meaningful characterization of the work force history.

The different sectors of the modern society are intricately interdependent. A change in one of them has immediate ramifications for others. This particularly applies to employment. The organized labor in one sector can precipitate temporary unemployment in others, when it votes to resort to strike action. Also the employment market is a highly competitive one. The supply of young inexperienced, highly qualified and sometimes overqualified people, racial and sex prejudices, mechanization, etc. result in some being hired, some early retired, and some fired.

In the developed societies, age 65 is considered as the age of retirement. In the developing nations, except for civil servants, there is no such concept of a retirement age. Persons have to work in order to survive. In some developed societies, some small segment of retired persons enter the labor force for personal and/or economic reasons. Others leave employment and later join the labor force for various other reasons (eg. to join a spouse who is transferred, or working elsewhere; join the graduate or technical school for higher education). In the actuarial type of methodology that is being employed in the construction of working life tables, the above discussed finer elements of labor force participation cannot be taken cognizance of. So we propose that the work force history of a person be looked at from a different perspective.

It is clear from the above discussion that the labor force history of a person can be characterized as a Markov Renewal process. Suitable modifications (eg. homogeniety assumption, use of mixing distributions) are needed to employ the model for a nation, or a large collectivity of people.

2 MARKOV RENEWAL PROCESS APPROXIMATION OF WORK FORCE HISTORY

At any point of time in one's life, one would be in one of the following states.

- S_0 -Not in the labor force by not having entered it
- S_1 -Employed
- S_2 -Unemployed for involuntary reasons
- S_3 -Unemployed for voluntary reasons (eg. to have a baby)
- S_4 -Retired

S_2 and S_3 are further divisible if detailed information is available. To make the state space complete, we introduce S_5 the death state.

The length of stay of a person in state S_i before moving to S_j is a random variable with a distribution function $F_{ij}(t)$. The transition from S_i to S_j, in the appropriate unit of time, is governed by the elements of a transition probability matrix (P_{ij}). A typical labor force history is shown in Fig. 1. The model suggested here is more comprehensive than the ones in Hoem (1976, 1977) and Hoem and Fong (1976). Hoem and Fong have the age factor brought into the model directly. That can be accomplished here also by dividing the state space on the basis of age.

Some remarks are in order now. Since the death state is an absorbing state, all the first passage times are infinite. But still demographically meaningful results can be developed. All the elements P_{i5} (i=0, 1, 2, 3, 4) are the mortality rates specific for the labor force status. If suitable information is at hand, this model can make use of the differential mortality by work force status. Obviously, for either sex a separate model needs to be constructed.

NOTATION

We use the accepted notation in developing the results

- T_{ij*} -wait in S_i before direct transition to S_j
- T_{ij} -first passage time from S_i to S_j
- $F_{ij}(t)$ -distribution function of the wait T_{ij*}
- μ_{ij*} -Mean of $F_{ij}(t)$
- μ_i $-\sum_j P_{ij}\mu_{ij*}$
- m_{ij} $-E(T_{ij})$

The mean first passage times from all states to the death state can be easily derived by employing the following result due to Barlow and Proschan

Theorem (Barlow-Proschan)

Let $[P, F(t)]$ be an absorbing semi-Markov process with k (0, 1, 2,...k-1) absorbing states where P has the normalized form $P=\begin{bmatrix} I & 0 \\ R & Q \end{bmatrix}$.

Then the mean time to absorption, starting in state i $(i > k)$ is $\sum_j m_{ij}\mu_j$ where $(m_{ij}) = (I-Q)^{-1}$

3 SOME DEMOGRAPHICALLY USEFUL RESULTS

These results are not of much interest to us. We derive some other useful results. Let r_{ij} be the expected time in state S_j before death given that the person started from the state S_i.

r_{ij} = E[time in S_j before death/start from S_i]

Then we have the following:

r_{11} =expected life time in employed status given that the person started from his/her first job

r_{12} =expected life time in unemployed state (after joining the work force) for involuntary reasons

r_{13} =expected life time in unemployed state (after joining the work force) for voluntary reasons

r_{14} =expected life time in retirement

$\sum_{i=1}^{4} r_{ij}$ is the total life time after joining the work force for the first time. Then

$r_{ij}/\sum_{i=1}^{4} r_{ij}$ is the fraction of one's labor force life spent in status S_i. These are of demographic interest.

Before we go on developing the results, a look at the transition matrix P is called for. For definitional reasons P should have the following form

$$P = \begin{array}{cccccc} S_0 & S_1 & S_2 & S_3 & S_4 & S_5 \end{array}$$
$$P = \begin{bmatrix} P_{00} & P_{01} & 0 & 0 & 0 & P_{05} \\ 0 & P_{11} & P_{12} & P_{13} & P_{14} & P_{15} \\ 0 & P_{21} & P_{22} & 0 & 0 & P_{25} \\ 0 & P_{31} & 0 & P_{33} & 0 & P_{35} \\ 0 & P_{41} & 0 & 0 & P_{44} & P_{45} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Let Y_{ij} be the random length of time in S_j before death given that the person started in S_i. For a first-step analysis, we have the following mutually exclusive possibilities:

a) the person can move directly from S_i $(i \neq 5)$ to the death state

b) the person can move from S_i to S_j directly and then from S_j to the death state

c) the person can move from S_i to S_k $(k \neq j, i)$ first and then from S_k to S_j from where the person moves to the death state

3.1 Theorem:

$$r_{11} = \mu_1 \Big/ 1 - \sum_{i=2}^{4} P_{1i} P_{i1}$$

Proof:

Now

$$Y_{11} = \begin{cases} T_{11*} & \text{Probability } P_{11} \\ T_{12*}+Y_{21} & \text{Probability } P_{12} \\ T_{13*}+Y_{31} & \text{Probability } P_{13} \\ T_{14*}+Y_{41} & \text{Probability } P_{14} \\ T_{15*} & \text{Probability } P_{15} \end{cases}$$

Then

$$r_{11} = E(Y_{11}) = \mu_1 + \sum_{i=2}^{4} P_{1i} r_{i1} \qquad (1)$$

To evaluate (1), we require the expressions for r_{21}, r_{31} and r_{41}.

Now

$$Y_{21} = \begin{cases} Y_{11} & \text{Probability } P_{21} \\ 0 & \text{Probability } 1-P_{21} \end{cases}$$

as the person has to move state from S_2 to S_1 with probability P_{21}. Then

$$r_{21} = P_{21}\, r_{11} \qquad (2)$$

Similarly

$$r_{31} = P_{31}\, r_{11} \qquad (3)$$

$$r_{41} = P_{41}\, r_{11} \qquad (4)$$

Substituting for r_{21}, r_{31}, and r_{41} from (2), (3), and (4), we get the expressions for r_{11} from (1).

3.2 Corollaries:

(a) $r_{21} = P_{21}\, r_{11}$

(b) $r_{31} = P_{31}\, r_{11}$

(c) $r_{41} = P_{41}\, r_{11}$

3.3 Theorem:

$$r_{12} = \frac{\mu_1 + P_{12}\mu_2 - P_{11}\mu_{11*}}{1 - \sum_{i=2}^{4} P_{1i} P_{i1}}$$

Proof:

$$Y_{12} = \begin{cases} T_{12*}+Y_{22} & \text{Probability } P_{12} \\ T_{13*}+Y_{32} & \text{Probability } P_{13} \\ T_{14*}+Y_{42} & \text{Probability } P_{14} \\ T_{15*} & \text{Probability } P_{15} \end{cases}$$

Then

$$r_{12} = (\mu_1 - P_{11}\mu_{11*}) + \sum_{i=2}^{4} P_{1i}\, r_{i2}$$

We have to evaluate r_{22}, r_{32} and r_{42};

$$Y_{22} = \begin{cases} T_{21*}+Y_{12} & \text{Probability } P_{21} \\ T_{22*} & \text{Probability } P_{22} \\ T_{25*} & \text{Probability } P_{25} \end{cases}$$

$$E(T_{22}) = r_{22} = \mu_2 + P_{21}\, r_{12}$$

Similarly

$$Y_{32} = \begin{cases} Y_{12} & \text{Probability } P_{31} \\ 0 & \text{Probability} \end{cases}$$

Thus $r_{32} = P_{31}\, r_{12}$

Similarly $r_{42} = P_{41}\, r_{12}$

$$\therefore r_{12} = (\mu_1 - P_{11}\mu_{11^*}) + P_{12}[\mu_2 + P_{21} r_{12}] + P_{13}P_{31} r_{12} + P_{14}P_{41} r_{12}$$

$$r_{12}\left[1 - \sum_{i=2}^{4} P_{1i}P_{i1}\right] = \mu_1 + P_{12}\mu_2 - P_{11}\mu_{11^*}$$

$$r_{12} = \frac{\mu_1 + P_{12}\mu_2 - P_{11}\mu_{11^*}}{1 - \sum_{i=2}^{4} P_{1i}P_{i1}}$$

3.5 Corollaries:

(a) $r_{32} = P_{31} r_{12}$

(b) $r_{42} = P_{41} r_{12}$

Similarly we have

3.6 Theorem:

$$r_{13} = \frac{\mu_1 + P_{13}\mu_3 - P_{11}\mu_{11^*}}{1 - \sum_{i=2}^{4} P_{1i}P_{i1}}$$

3.7 Theorem:

$$r_{14} = \frac{\mu_1 + P_{14}\mu_4 - P_{11}\mu_{11^*}}{1 - \sum_{i=2}^{4} P_{1i}P_{i1}}$$

3.8 General Remarks:

The results derived above do not consider the effects of age, sex, education, etc. on work force history. These variables can be easily incorporated into the model by increasing the state space on the bases of these characteristics. Each of our S_i ($i = 0, 1, \ldots 5$) could be thought of as states for each of the age groups for either sex and the various educational categories. The transition matrix would have then a large number of zero enteries.

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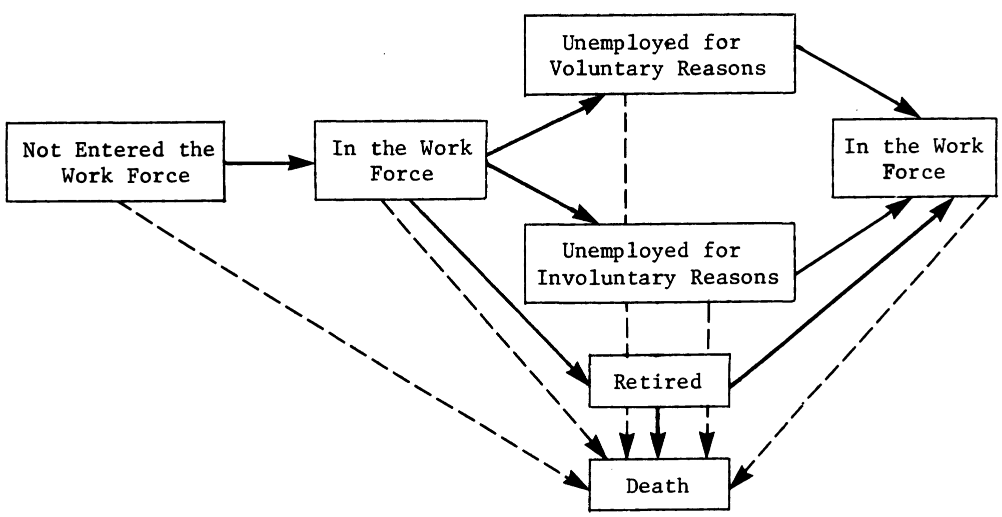

FIG 1A. A TYPICAL WORK FORCE HISTORY (Flow Chart)

Legend: S_0 - Not entered the work force

S_1 - In the work force

S_2 - Unemployed for involuntary reasons

S_3 - Unemployed for voluntary reasons

S_4 - Retired

S_5 - Death

FIG 1B. A TYPICAL WORK FORCE HISTORY